

حلول التمارين حول القوى

<p style="text-align: right;">(9)</p> $\begin{aligned} (2-3(2-3)^{-1})^{-1} &= [2-3(-1)^{-1}]^{-1} \\ &= \left[2-3\left(\frac{1}{-1}\right)\right]^{-1} \\ &= [2-3(-1)]^{-1} \\ &= [2+3]^{-1} \\ &= 5^{-1} \\ &= \frac{1}{5} \end{aligned}$ <p>إذن الإجابة الصحيحة هي $\frac{1}{5}$</p> <p>(10)</p> $\begin{aligned} B &= (3x-7)^2 & A &= (2x+3)^2 \\ &= (3x)^2 - 2 \times 3x \times 7 + 7^2 & &= (2x)^2 + 2 \times 2x \times 3 + 3^2 \\ &= 9x^2 - 42x + 49 & &= 4x^2 + 12x + 9 \end{aligned}$ $\begin{aligned} D &= (2x^2+5)(2x^2-5) & C &= \left(x-\frac{2}{3}\right)\left(x+\frac{2}{3}\right) \\ &= (2x^2)^2 - 5^2 & &= x^2 - \left(\frac{2}{3}\right)^2 \end{aligned}$	<p>(1)</p> $\begin{aligned} (-5)^3 &= -125 & , & (-2)^5 = -32 & , & 2^3 = 8 \\ 0^{20} &= 0 & , & (-1)^{112} = 1 & , & 1^{75} = 1 \end{aligned}$ <p>(2)</p> $\begin{aligned} B &= [2 \times (-5)]^2 & A &= (-2)^3 \times (-3)^2 \\ &= (-10)^2 & &= -8 \times 9 \\ &= 100 & &= -72 \end{aligned}$ $\begin{aligned} C &= [(-1)^{17} \times (-2)^3]^2 \\ &= [-1 \times (-8)]^2 \\ &= 8^2 \\ &= 64 \end{aligned}$ $\begin{aligned} D &= [2 \times (-3)^2] \times [4 \times (-5)^2]^2 \\ &= (2 \times (-3)^2) \times (4 \times (-5)^2)^2 \\ &= (2 \times 9) \times (4 \times 25)^2 \\ &= 18 \times 100^2 \\ &= 180000 \end{aligned}$
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$$= 4x^4 - 25 \quad = x^2 - \frac{4}{9}$$

$$x^2 + 8x + 16 = (x + 4)^2 \quad (11)$$

$$(3x-2)^2 = 9x^2 - 12x + 4$$

$$4x^2 - \frac{25}{9} = \left(2x + \frac{5}{3}\right)\left(2x - \frac{5}{3}\right)$$

$$x^2 + x + \frac{1}{4} = \left(x + \frac{1}{2}\right)^2$$

$$B = x^2 - \frac{1}{2}x + \frac{1}{16} \quad A = 4a^2 - 49 \quad (12)$$

$$= x^2 - 2 \cdot \frac{1}{4} \cdot x + \left(\frac{1}{4}\right)^2 = (2a)^2 - 7^2$$

$$= \left(x - \frac{1}{4}\right)^2 = (2a + 7)(2a - 7)$$

$$\begin{aligned} C &= (x^2 - 4) + (x - 2)(5x + 3) \\ &= (x - 2)(x + 2) + (x - 2)(5x + 3) \\ &= (x - 2)[(x + 2) + (5x + 3)] \\ &= (x - 2)(6x + 5) \end{aligned}$$

$$\begin{aligned} D &= 3(x-5)^2 - 2(x^2 - 25) \\ &= 3(x-5)(x-5) - 2(x-5)(x+5) \\ &= (x-5)[3(x-5) - 2(x+5)] \\ &= (x-5)(3x-15-2x-10) \\ &= (x-5)(x-25) \end{aligned}$$

(13) أ) إذا كان n زوجيا فإن $n+1$ فردي،
ومنه :

$$\begin{aligned} A &= (-1)^n + (-1)^{n+1} \\ &= 1 + (-1) \\ &= 0 \end{aligned}$$

$$\begin{aligned} B &= 3^2(-1)^n - (-2)^2(-1)^{n+1} \\ &= 3^2 \times 1 - (-2)^2(-1) \\ &= 9 + 4 \\ &= 13 \end{aligned}$$

ب - إذا كان n فرديا فإن $n+1$ زوجي ومنه :

$$B = \left(\frac{-16 \times 3^2}{24 \times (-3)}\right)^{-1} \quad A = \left(\frac{2}{3}\right)^{-1} \times \left(\frac{-3}{4}\right)^{-1} \quad (3)$$

$$\begin{aligned} &= \frac{24 \times (-3)}{-16 \times 3^2} = \frac{3}{2} \times \frac{-4}{3} \\ &= \frac{-8 \times 3 \times 3}{-8 \times 2 \times 3 \times 3} = \frac{-4}{2} \\ &= \frac{-1}{-2} = \frac{1}{2} = -2 \end{aligned}$$

$$\begin{aligned} A &= (-3)^5 \times (-3)^7 \times (-3)^{-11} \quad (4) \\ &= (-3)^{5+7-11} \\ &= (-3)^1 \\ &= -3 \end{aligned}$$

$$B = \frac{(-5)^2 \times (25)^{-3}}{5^3 \times (25)^{-2}}$$

$$\text{لاحظ أن } = \frac{(-5)^2 \times (5^2)^{-3}}{5^3 \times (5^2)^{-2}}$$

$$(-5)^2 = 5^2 \text{ و } 25 = 5^2$$

$$\begin{aligned} &= \frac{5^2 \times 5^{-6}}{5^3 \times 5^{-4}} \\ &= \frac{5^{2-6}}{5^{3-4}} \\ &= \frac{5^{-4}}{5^{-1}} \\ &= 5^{-4+1} \\ &= 5^{-3} \\ &= \frac{1}{5^3} \\ &= \frac{1}{125} \end{aligned}$$

$$A = \frac{a^2 b^3}{a^3 b^2} = \frac{a^2 b^2 b}{a^2 a b^2} = \frac{b}{a} \quad (5)$$

$$B = \frac{(2a^2 \times b^3)^3}{(3ab^4)^2} = \frac{2^3 a^6 b^9}{3^2 a^2 b^8} = \frac{8a^4 b}{9}$$

$$\begin{aligned} A &= (-1)^n + (-1)^{n+1} \\ &= -1 + 1 \\ &= 0 \end{aligned}$$

$$\begin{aligned} B &= 3^2(-1)^n - (-2)^2(-1)^{n+1} \\ &= 3^2 \times (-1) - (-2)^2(1) \\ &= -9 - 4 \\ &= -13 \end{aligned}$$

ملاحظة : في كلتا الحالتين $A = 0$ ونتيجتي B

متقابلتين.

(14) نبسط أولا a و b باستعمال التعميل

$$\begin{aligned} a &= 2^{n-1} + 2^n + 2^{n+1} \\ &= 2^{n-1}(1 + 2 + 2^2) \\ &= 2^{n-1}(7) \\ &= 7 \times 2^{n-1} \\ b &= 2^{n-2} + 2^{n-1} + 2^n \\ &= 2^{n-2}(1 + 2 + 2^2) \\ &= 2^{n-2}(7) \\ &= 7 \times 2^{n-2} \end{aligned}$$

$$\begin{aligned} a^2 &= (7 \times 2^{n-1})^2 \\ &= 7^2 \times (2^{n-1})^2 \\ &= 49 \times 2^{2n-2} \\ b^2 &= (7 \times 2^{n-2})^2 \\ &= 7^2 \times (2^{n-2})^2 \\ &= 49 \times 2^{2n-4} \end{aligned}$$

$$\begin{aligned} c^2 &= (\sqrt{147} \times 2^{n-2})^2 \\ &= 147 \times 2^{2n-4} \end{aligned}$$

$$\begin{aligned} b^2 + c^2 &= 49 \times 2^{2n-4} + 147 \times 2^{2n-4} \quad \text{و منه} \\ &= (49 + 147) 2^{2n-4} \\ &= 196 \times 2^{2n-4} \\ &= 49 \times 4 \times 2^{2n-4} \\ &= 49 \times 2^2 \times 2^{2n-4} \\ &= 49 \times 2^{2n-2} \end{aligned}$$

لدينا $b^2 + c^2 = a^2$ إذن : المثلث ABC قائم الزاوية في A.

(15)

$$^{+1}.11^{3k+1}.5^{3k} + 539 = 7 \times 7^{3k}.11 \times 11^{3k}.5^{3k} + 7 \times 11 \times 7$$

$$\begin{aligned} C &= \frac{a^2 b^3}{(ab)^3} \div \left(\frac{a^2 b^4}{a^3 b^5} \right)^{-1} = \frac{a^2 b^3}{(ab)^3} \div \frac{a^3 b^5}{a^2 b^4} \\ &= \frac{a^2 b^3}{a^3 b^3} \times \frac{a^2 b^4}{a^3 b^5} = \frac{a^2 b^3 a^2 b^4}{a^3 b^3 a^3 b^5} = \frac{a^4 b^7}{a^6 b^8} = \frac{1}{a^2 b} \end{aligned}$$

$$A = (ab)^2 \left(\frac{a^2}{b^2} + \frac{b^2}{a^2} \right) \quad (6)$$

$$\begin{aligned} &= a^2 b^2 \left(\frac{a^4 + b^4}{a^2 b^2} \right) \\ &= a^4 + b^4 \end{aligned}$$

$$\begin{aligned} B &= \frac{(ab^3)^2}{a^3 b^2} \left[\frac{a}{b^4} + \frac{a^2}{b^3} \right] \\ &= \frac{a^2 b^6}{a^3 b^2} \left(\frac{a + a^2 b}{b^4} \right) \\ &= \frac{a^2 b^6 (a + a^2 b)}{a^3 b^2 b^4} \\ &= \frac{a^2 b^6 a(1 + ab)}{a^3 b^6} \\ &= \frac{a^3 b^6 (1 + ab)}{a^3 b^6} \\ &= 1 + ab \end{aligned}$$

$$\begin{aligned} A &= 2,3 \times 10^2 + 5,28 \times 10^{-1} \quad (7) \\ &= 230 + 0,528 \\ &= 230,528 \\ &= 2,30528 \times 10^2 \end{aligned}$$

$$\begin{aligned} B &= (53,27 \times 10^{-2}) \div 20 \\ &= 0,5327 \div 20 \\ &= 0,026635 \\ &= 2,6635 \times 10^{-2} \end{aligned}$$

$$\begin{aligned} C &= \frac{45 \times 10^{-2}}{12 \times 10^{-3}} \\ &= \frac{45}{12} \times 10^{-2+3} \\ &= \frac{15}{4} \times 10^1 \\ &= 3,75 \times 10 \end{aligned}$$

$$\begin{aligned}
&= 7 \times 11 \times (7^{3k} \cdot 11^{3k} \cdot 5^{3k} + 7) \\
&= 77(7 \times 7^{3k-1} \cdot 11^{3k} \cdot 5^{3k} + 7) \\
&= 77 \times 7(7^{3k-1} \cdot 11^{3k} \cdot 5^{3k} + 1) \\
&= 539(7^{3k-1} \cdot 11^{3k} \cdot 5^{3k} + 1)
\end{aligned}$$

وبملاحظة أن $7^{3k-1} \cdot 11^{3k} \cdot 5^{3k}$ عدد فردي (جداء أعداد فردية) فإن $7^{3k-1} \cdot 11^{3k} \cdot 5^{3k} + 1$ عدد زوجي أي $7^{3k+1} \cdot 11^{3k+1} \cdot 5^{3k} + 539 = 539(2k')$ $(k' \in \mathbb{Z})$
 $= 1078k'$

إذن $7^{3k+1} \cdot 11^{3k+1} \cdot 5^{3k} + 539$ مضاعف للعدد 1078

$$\left(x^{-1} + y^{-1}\right)^{-1} = \left(\frac{1}{x} + \frac{1}{y}\right)^{-1} \quad (8)$$

$$= \left(\frac{y+x}{xy}\right)^{-1}$$

$$= \left(\frac{x+y}{xy}\right)^{-1}$$

$$= \frac{xy}{x+y}$$

إذن الإجابة الصحيحة هي $\frac{xy}{x+y}$